Division of Strength of Materials and Structures
Faculty of Power and Aeronautical Engineering

Finite element method (FEM1)

Formulas for Test 1

04.2025

Rules for writing Test 1

To write efficiently and not waste time:

- We enter the room only with writing utensils in hand,
- When entering the room, please leave your bags against the wall or on the windows,
- Take your seats efficiently (every second chair in a row).

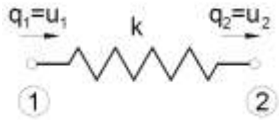
Group A

Enters the room from 12:00
Distribution of questions 12.05
Test starts 12.10
End of writing 13.00
Quick exit from the room

Grupa B

Enters the room from 13.05
Distribution of questions 13.10
Test starts 13.15
End of writing 14.05
Quick exit from the room

Local spring stiffness matrix:

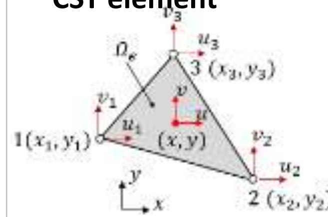


$$[k]_e = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

$$[A]^{-1} = \frac{1}{\det A}[\alpha_{ik}]^T$$

where α_{ik} are the algebraic complements of the elements a_{ik} of the matrix [A].

CST element



shape functions =normalized area coordinates:

$$N_1(x,y) = L_1(x,y) = \frac{A_1(x,y)}{A_e} = \frac{1}{2A_e}(a_1 + b_1 x + c_1 y)$$

$$N_2(x,y) = L_2(x,y) = \frac{A_2(x,y)}{A_e} = \frac{1}{2A_e}(a_2 + b_2 x + c_2 y)$$

$$N_3(x,y) = L_3(x,y) = \frac{A_3(x,y)}{A_e} = \frac{1}{2A_e}(a_3 + b_3 x + c_3 y)$$

$a_1 = x_2 y_3 - x_3 y_2$; $a_2 = x_3 y_1 - x_1 y_3$; $a_3 = x_1 y_2 - x_2 y_1$

$b_1 = y_2 - y_3$; $b_2 = y_3 - y_1$; $b_3 = y_1 - y_2$

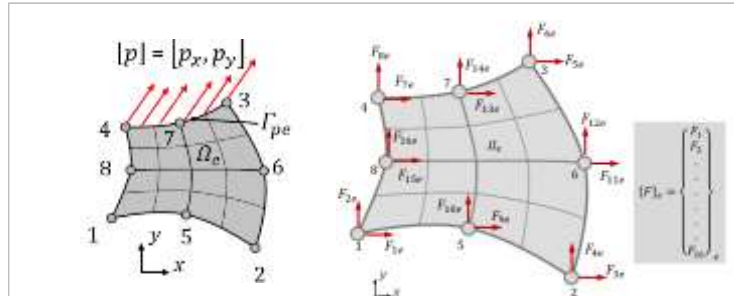
$c_1 = x_3 - x_2$; $c_2 = x_1 - x_3$; $c_3 = x_2 - x_1$

equivalent load vector due to surface load:

$$\lfloor F^p \rfloor_e = t_e \int_0^l \lfloor p \rfloor [N]\, ds$$

equivalent load vector due to mass forces:

$$\lfloor F^X \rfloor_e = t_e \int_{A_e} \lfloor X \rfloor [N] dA_e$$



Shape functions of the 8-node quadrilateral finite elment:

$$N_1(\xi,\eta) = -\frac{1}{4}(1-\xi)(1-\eta)(1+\xi+\eta)$$

$$N_2(\xi,\eta) = -\frac{1}{4}(1+\xi)(1-\eta)(1-\xi+\eta)$$

$$N_3(\xi,\eta) = -\frac{1}{4}(1+\xi)(1+\eta)(1-\xi-\eta)$$

$$N_4(\xi,\eta) = -\frac{1}{4}(1-\xi)(1+\eta)(1+\xi-\eta)$$

$$N_5(\xi,\eta) = \frac{1}{2}(1-\xi^2)(1-\eta)$$

$$N_6(\xi,\eta) = \frac{1}{2}(1+\xi)(1-\eta^2)$$

$$N_7(\xi,\eta) = \frac{1}{2}(1-\xi^2)(1+\eta)$$

$$N_8(\xi,\eta) = \frac{1}{2}(1-\xi)(1-\eta^2)$$

Integrals of barycentric functions:

$$J_1 = \int_0^l L_1^q L_2^r\, dl = \frac{q!\,r!\,l}{(q+r+1)!}$$

$$J_2 = \int_{A_e} L_1^q L_2^r L_3^t dA_e = \frac{q!\,r!\,t!}{(q+r+t+2)!} 2A_e$$

shape function matrix:

$$\underset{2\times 16}{[N(\xi,\eta)]} = \begin{bmatrix} N_1(\xi,\eta) & 0 & N_2(\xi,\eta) & 0 & \dots & N_8(\xi,\eta) & 0 \\ 0 & N_1(\xi,\eta) & 0 & N_2(\xi,\eta) & \dots & 0 & N_8(\xi,\eta) \end{bmatrix}$$

Gaussian quadrature rule:

$$\int_{-1}^{1} f(\xi) d\xi = \sum_{i=1}^{n} w_i \cdot f(\xi_i) + R_n$$

$$R_n = 0 \quad \Rightarrow \quad \frac{d^{2n} f}{d\xi^{2n}} = 0$$

For 2D elements:

$$\int_{-1}^{1}\int_{-1}^{1} f(\xi,\eta)\, d\xi d\eta = \sum_{j=1}^{n}\sum_{i=1}^{n}(w_i w_j \cdot f(\xi_i, \eta_j))$$

Polynomial degree	Number of Gauss points	ξ_i	w_i
1	1	0	2
3	2	$-1/\sqrt{3}$ $+1/\sqrt{3}$	1 1
5	3	$-\sqrt{0.6}$ 0 $+\sqrt{0.6}$	5/9 8/9 5/9